6. SELECTED APPLICATIONS

6.1 TRUSSES

6.1.1 **Definition of a truss**. The truss is one of the major types of engineering structures. It provides both a practical and economical solution to many engineering situations, especially in the design of bridges and buildings. A truss consists of straight members connected at joints; a typical truss is shown in Fig.6.1. Truss members are connected at their extremities only; thus no member is continuous through a joint. In Fig.6.1, for example, there is no member *AB*; there are instead two distinct members, *AD* and *DB*. Actual structures are made of several trusses joined together to form a space framework. Each truss is designed to carry those loads which act in its plane and thus may be treated as a two-dimensional structure.

In general, the members of a truss are slender and can support little lateral load; all loads, therefore, must be applied to the various joints, and not to members themselves. When a concentrated load is to be applied between two joints, or when a distributed load is to be supported by the truss, as in the case of a bridge truss, a floor system must be provided which, through the use of strings and floor beams, transmits the load to the joints.

The weights of the members of the truss are also assumed to be applied to the joints, half of the weight of each member being applied to each of the two joints the member connects. Although the members are actually joined together by means of riveted and welded connections, it is customary to assume that the members are pinned together; therefore, the forces acting at each end of a member reduce to a single force and no couple. Thus, the only forces assumed to be applied to a truss member are single force at each end of the member. Each member may then be treated as a two-force member, and the entire truss may be considered as a group of pins and two-force members (Fig.6.1). An individual member may be acted upon as shown in either of the two sketches of Fig.6.2. In the first sketch, the forces tend pull the member apart, and the member is in tension, while, in the second sketch, the forces tend to compress the member, and the member is in compression. Several typical trusses are shown in Fig.6.3.

6.1.2 **Simple trusses**. Consider the truss of Fig.6.4a, which is made of four members connected by pins at *A*, *B*, *C* and *D*. If a load is applied at *B*, the truss will greatly deform and lose completely its original shape. On the other hand, the truss of Fig.6.4b, which is made of three members connected by pins at *A*, *B* and *C*, will deform only slightly under a load applied at *B*. The only possible deformation for this truss is one involving small changes in the length of its members. The truss of Fig.6.4b is said to be a *rigid truss*, the term rigid being used here to indicate that the truss *will not collapse.*

As shown in Fig.6.4c, a larger rigid truss may be obtained by adding two members *BD* and *CD* to the basic triangular truss of Fig.6.4b. The procedure may be repeated as many times as desired, and the resulting truss will be rigid if, each time we add two new members, we attach them to separate existing joints and connect them at a new joint (the three joints must not be in a straight line). A truss which may be constructed in this manner is called a *simple truss*.

The basic triangular truss of Fig.6.4b has three members and three joints. The truss of Fig.6.4c has two more members and one more joint, i.e. five members and four joints. Observing that every time two new members are added, the number of joints is increased by one, we find that in a simple truss the total number of members is

$$m = 2n - 3 \qquad (6.1a)$$

where n is the total number of joints.

The analogous relation for space trusses is as follows

$$m = 3n - 6 \qquad (6.1b)$$

It should be noted that a simple truss is not necessarily made only of triangles. The truss of Fig.6.4d, for example, is a simple truss which was constructed from a triangle *ABC* by adding successively the joints *D, E, F* and *G.*

6.1.3 **Analysis of trusses by the method of joints**. We saw in Sec. 6.1.1 that a truss may be considered as a group of pins and two-force members. The truss of Fig.6.1 whose free-body diagram is shown in Fig.6.5a, may thus be dismembered, and a free-body diagram can be drawn for each pin and each member (Fig.6.5b). Each member is acted upon by two forces, one at each end; these forces have the same magnitude, same line of action, and opposite sense. Besides, Newton's third law indicates that the forces of action and reaction between a member and a pin are equal and opposite. Therefore, the forces exerted by a member on the two pins it connects must be directed along that member and be equal and opposite. The common magnitude of the forces exerted by a member on the two pins it connects is commonly referred to as the *force in the member* considered, even though this quantity is actually a scalar. Since the lines of action of all the internal forces in a truss are known, the analysis of a truss reduces to the computation of the forces in its various members and to the determination of whether each of its members is in tension or in compression.

Since the entire truss is in equilibrium, each pin must be in equilibrium. The fact that a pin is in equilibrium may be expressed by drawing its free-body diagram and writing two equilibrium equations. If the truss contains n pins, there will be therefore $2n$ equations available, which may be solved for $2n$ unknowns. In the case of simple truss, we have $m = 2n - 3$, that is, $2n = m + 3$, and the number of unknowns which may be determined from the free-body diagrams of the pins is thus $m + 3$. This means that the forces in all the members, as well as the three external unknowns may be found by considering the free-body diagrams of the pins. In the case under consideration of the truss from Fig.6.5a these are two components of the reaction $\mathbf{R}_A$, and the reaction $\mathbf{R}_B$.

The fact that the entire truss is a rigid body in equilibrium may be used to write three more equations involving the forces shown in the free-body diagram of Fig.6.5a. Since they do not contain any new information, these equations are not independent from the equations associated with free-body diagrams of the pins. Nevertheless, they may be used to determine immediately the components of the reactions at the supports. The arrangement of pins and members in a simple truss is such that it will then always be possible to find a joint involving only two unknown forces. These forces may be determined by either analytical or geometrical methods and their values transferred to the adjacent joints and treated as known quantities at these joints. This procedure may be repeated until all unknown forces have been determined.

An example of the analysis concerning a determination of forces in truss members will be provided at the lecture.

Bibliography

Beer F.P, Johnston E.R., Jr., Vector Mechanics for Engineers, McGraw-Hill